# Discrete Structures (2)
# Course Logistics and Introduction

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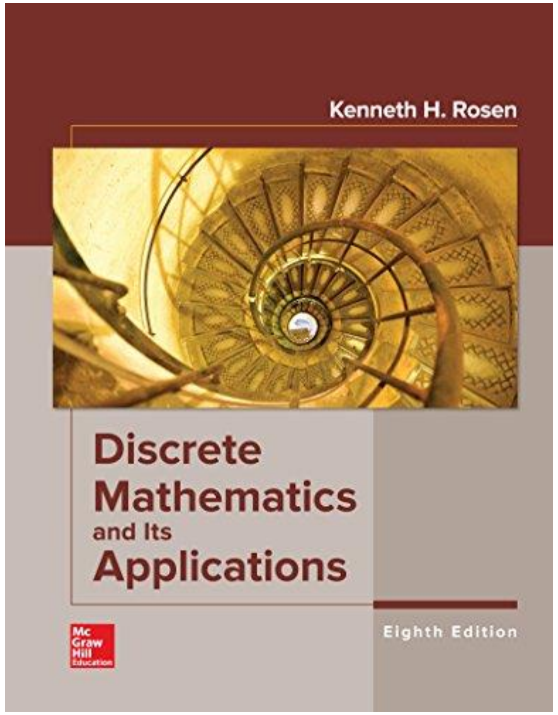
# Course Information

- **14011801-3: Discrete Structures (2)**
  - Contact hours: Lecture 2 Hours, Tutorial 2 Hours.
  - Prerequisites: 14011801-3 Discrete Structures I & 14011101-4 Computer Programming
- **CS2102: Discrete Structures (2)**
  - Contact hours: Lecture 2 Hours, Tutorial 2 Hours.
  - Prerequisites: CS1101 Discrete Structures (1)




- Main Text Book: Rosen, *Discrete Mathematics & its Applications* ($8^{th}$ edition).
- Course Website: Course page at BlackBoard
- Lectures Notes : They can be downloaded from the website in PDF format.

# Grading Scheme

- 10% Weekly Assignments [Individual work]
  - a total of 10-8 assignments. One per week.
- 20% Quizzes [Individual work]
  - a total of 4 quizzes, a quiz per topic. Best 3 are counted.
- 20% Midterm [Individual work]
- 50% Final Exam [Individual work]
- 3% Bonus: Mini-projects [Group work]
  - Choose two mini programming projects from the given list.

# Tutorial Assignments

- The weekly tutorial is to demonstrate your understanding of the concepts you learned during the lectures.

- You must form a **team of pairs (2 students)** from the beginning of the term. There will be several questions to be solved in the tutorial sessions.

- Each week, there will be an untimed online quiz on the blackboard with unlimited attempts and worth 1%.

- Weekly assignments are considered individual assessment; however, you are allowed to work on them with your teammate.

# Class, Quiz and Exam Policies

- Please be at the class on time. Latecomers will not be allowed to attend the class.
- Please turn off your mobiles while you are in the class. You would be forced to leave the class if your mobile rang.
- Any absence from any assessment (i.e., quizzes, tutorial assignments, exams) would make you lose their marks unless you got an official excuse.
- Each student is expected to attend all lectures. Any absence without an excuse will decrease your total grade. (Each absence will be counted as a one-half lost mark).

CH6

# COUNTING BASICS

قواعد العد

Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# AGENDA

- Basic Counting Principles:
  - The Product Rule
  - The Sum Rule
  - The Subtraction Rule
  - The Division Rule
- Tree Diagrams
- The Pigeonhole Principle
  - The Generalized Pigeonhole Principle

# Introduction

- **Counting problems** are of the following kind:
  - **How many** different 8-letter passwords are there?
  - **How many** possible ways are there to pick 11 football players out of a 20-player team?
  - **How many** times a step in an algorithm is executed?
  - **How many** IP addresses are there?

Most importantly, counting is the basis for computing probabilities of discrete events. ("**What is the probability** of winning the lottery?")

العد هو اساس حساب الاحتمالات لمجموعه من الاحداث المنفصلة

# Basic Counting Principles

- Counting problems may be very hard and not obvious. تفكيك
  - Solution: Break down the problem into smaller pieces (decomposition)

- Two basic **counting principles**: يتم تقسيم المسألة الى مسائل مرتبطة
  - قاعدة الضرب **Product rule**: the problem is decomposed into **dependent** counts. $\times$ $m \times n$

    Total counts = product of all counts

    مستقلة
  - قاعدة الجمع **Sum rule**: the problem is decomposed into **independent** counts. $m+n$

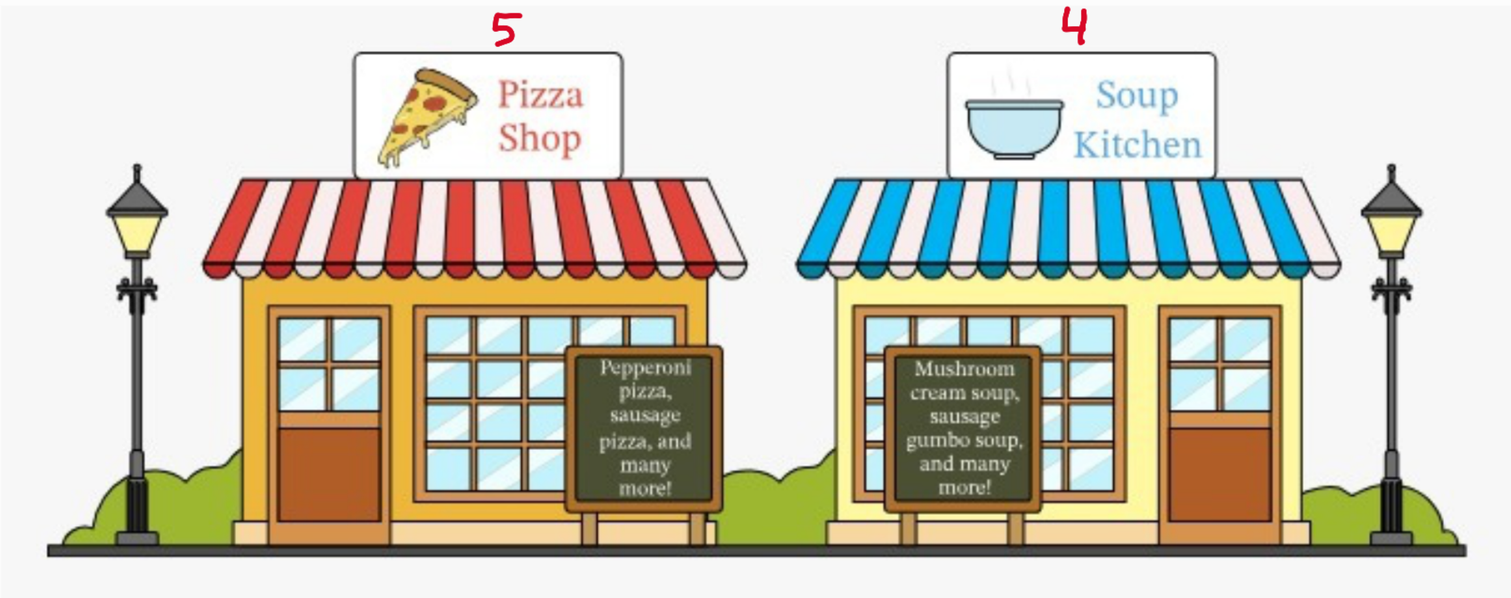
    Total counts = sum of all counts $+$

# Product rule

dependent


# Sum rule

independent




We have 5 + 4 = 9 different options for lunch

قاعدة الضرب

# The Product Rule

$n_1 \times n_2$

**The Product Rule**: A procedure can be broken down into a sequence of two tasks. There are $n_1$ ways to do the first task and $n_2$ ways to do the second task. Then there are $n_1 \cdot n_2$ ways to do the procedure.

First task $n_1$ second tansk $n_2$

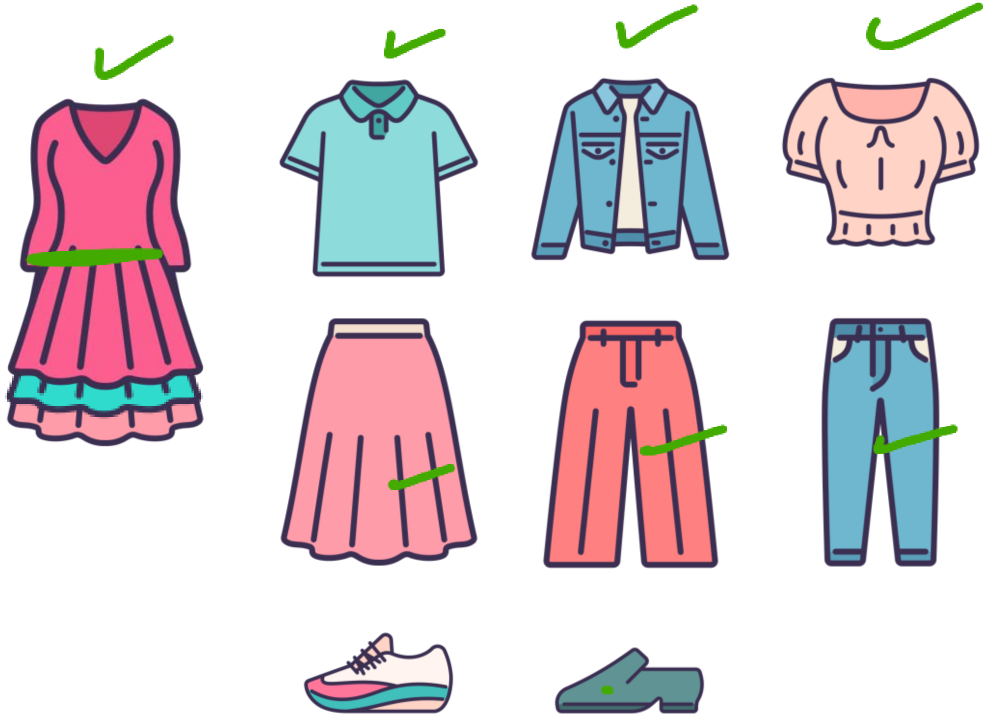
تعميم

**Generalized product rule:** If we have a procedure consisting of sequential tasks $T_1, T_2, \ldots, T_m$ that can be done in $n_1, n_2, \ldots, n_m$ ways, respectively, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_m$ ways to carry out the procedure.

$$\begin{matrix} T_1 & T_2 & T_3 & \cdots & T_m \\ n_1 & n_2 & n_3 & & n_m \end{matrix} = n_1 \cdot n_2 \cdot n_3 \cdots n_m$$

# The Product Rule: Examples

Example: How many outfits cam you have using the following items?



Solution:

#options for the top: 4

#options for the bottom: 4

#options for the shoes: 2

Then using the product rule,

We can have $4 \cdot 4 \cdot 2 = 32$ outfits.

# The Product Rule: Examples

Example: How many bit strings of length seven are there?

Solution: Since each of the seven bits is either a $0$ or a $1$, the answer is $2^7 = 128$.

$$
\begin{array}{ccccccc}
1 & 1 & & & & & \\
0 & 0 & & & & & \\
\hline
\end{array}
$$

$$n = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$= 2^7 = 128$$

# The Product Rule: Examples

لوحة السيارة

**Example:** How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?

0 1 2 3 4 5 6 7 8 9

**Solution:** By the product rule,

there are 26 · 26 · 26 · 10 · 10 · 10 = 17,576,000 different possible license plates.

$$\underbrace{26 \cdot 26 \cdot 26}_{\text{حروف}} \cdot \underbrace{10 \cdot 10 \cdot 10}_{\text{أرقام}}$$

$$\underbrace{\_\_\ \_\_\ \_\_}_{\text{26 choices for each letter}} \quad \underbrace{\_\_\ \_\_\ \_\_}_{\text{10 choices for each digit}}$$

# The Product Rule: Examples

**Example:** The *North American numbering plan* (*NANP*) specifies that a telephone number consists of 10 digits, consisting of a three-digit area code, a three-digit office code, and a four-digit station code. There are some restrictions on the digits.

- Let $X$ denote a digit from 0 through 9. $= 10$
- Let $N$ denote a digit from 2 through 9. $= 8$
- Let $Y$ denote a digit that is 0 or 1. $= 2$
- In the old plan (in use in the 1960s) the format was *NYX-NNX-XXXX*.
- In the new plan, the format is *NXX-NXX-XXXX*.

كود المنطقة — — —   كود مكتب — — —   كود محطة — — — —

How many different telephone numbers are possible under the old plan and the new plan?

old plan

$$\underset{8}{N}\,\underset{2}{Y}\,\underset{10}{X} - \underset{8}{N}\,\underset{8}{N}\,\underset{10}{X} - \underset{10}{X}\,\underset{10}{X}\,\underset{10}{X}\,\underset{10}{X}$$

$$8\cdot 2\cdot 10\cdot 8\cdot 8\cdot 10\cdot 10\cdot 10\cdot 10\cdot 10 = 8^3\cdot 10^6\cdot 2$$

$$1{,}024{,}000{,}000$$

New Plan

$$\underset{8}{N}\,\underset{10}{X}\,\underset{10}{X} - \underset{8}{N}\,\underset{10}{X}\,\underset{10}{X} - \underset{10}{X}\,\underset{10}{X}\,\underset{10}{X}\,\underset{10}{X}$$

$$10^8\cdot 8^2 = 6{,}400{,}000{,}000$$

# The Product Rule: Examples (con.)

## Solution:

Use the Product Rule.

- There are $8 \cdot 2 \cdot 10 = 160$ area codes with the format *NYX.*
- There are $8 \cdot 10 \cdot 10 = 800$ area codes with the format *NXX.*
- There are $8 \cdot 8 \cdot 10 = 640$ office codes with the format *NNX.*
- There are $10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$ station codes with the format *XXXX.*

Number of old plan telephone numbers: $160 \cdot 640 \cdot 10{,}000 = 1{,}024{,}000{,}000$.

Number of new plan telephone numbers: $800 \cdot 800 \cdot 10{,}000 = 6{,}400{,}000{,}000$.

# The Product Rule: Examples

What is the value of $k$ after the following code, where $n_1, n_2, \ldots, n_m$ are positive integers, has been executed?

```
k := 0
for i₁ := 1 to n₁
    for i₂ := 1 to n₂
          .
          .
          .
        for iₘ := 1 to nₘ
          k := k+1
```

We use the product rule because for iteration in outer loop $n_1$ we do $n_2$ inner loops and so on.

So, $k = n_1 n_2 \ldots n_m$

# The Sum Rule

او

**The Sum Rule**: If a task can be done either in one of $n_1$ ways or in one of $n_2$, where none of the set of $n_1$ ways is the same as any of the $n_2$ ways, then there are $n_1 + n_2$ ways to do the task.

$T = n_1 + n_2$

تعميم

**Generalized sum rule:** If we have tasks $T_1, T_2, \ldots, T_m$ that can be done in $n_1, n_2, \ldots, n_m$ ways, respectively, and no two of these tasks can be done at the same time, then there are $n_1 + n_2 + \ldots + n_m$ ways to do one of these tasks.

لايمكن عمل اثنين منها بنفس الوقت ( مستقلة )

# The Sum Rule: Examples

**Example**: The CS department must choose either a student or a faculty member as a representative for a university committee. How many choices are there for this representative if there are 37 members of the CS faculty and 83 CS majors and no one is both a faculty member and a student.

ماعدد طرق اختيار الطالب او موظف في الكليه ليمثل في لجنه الجامعة

**Solution**: By the sum rule it follows that there are 37 + 83 = 120 possible ways to pick a representative.

$$83 + 37 = 120$$

# The Sum Rule: Examples

What is the value of $k$ after the following code, where $n_1, n_2, \ldots, n_m$ are positive integers, has been executed?

```
k := 0
for i₁ := 1 to n₁
    k := k+1
for i₂ := 1 to n₂
    k := k+1
            .
            .
            .
for iₘ := 1 to nₘ
    k := k+1
```

We use the sum rule because loops do not depend on each other.

So $k = n_1 + n_2 + \ldots + n_m$

A B

$A \cap B = \phi$
disjoint

A B $A \cap B \neq \phi$

قاعده الجمع في المجموعات

# The Sum Rule in terms of sets.

or
أو
$\cup$
$+$

- The sum rule can be phrased in terms of sets.

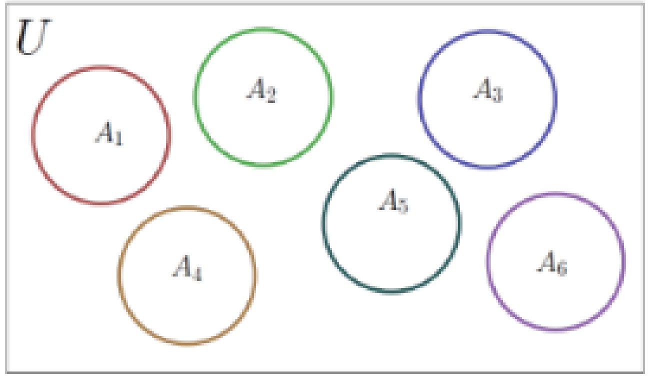
  $|A \overset{+}{\cup} B| = |A| + |B|$ as long as $A$ and $B$ are disjoint sets.

- Or more generally,

  $$|A_1 \cup A_2 \cup \cdots \cup A_m| = |A_1| + |A_2| + \cdots + |Am|$$

  when $A_i \cap A_j = \emptyset$ for all $i,j$.

اتحاد مجموعتين = جمعهما في حاله عدم التقاطع



# Combining the Sum and Product Rule

Combining the sum and product rule allows us to solve more complex problems.

**Example**: Suppose naming labels in a programming language can be either a single letter or a letter followed by a digit. Find the number of possible labels.

**Solution**: Let L be the total number of labels and $L_1$ and $L_2$ are the number of labels of length 1 and 2.

By the sum rule: $L = L_1 + L_2$

By product rule: $L_1 = 26$ $L_2 = 26 \cdot 10 = 260$

So $L = L_1 + L_2 = 26 + 260 = 286$

طريقة التسمية

حرف or $\underset{\text{حرف}}{26}$ $\underset{\text{رقم}}{10}$

$L_1$ $L_2$

$L = L_1 + L_2$ = عدد طرق التسميه

$= 26 + 26 \times 10 = 286$

# Counting Passwords

**Example**: Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

**Solution**: Let P be the total number of passwords, and let $P_6$, $P_7$, and $P_8$ be the passwords of length 6, 7, and 8.

- By the sum rule $P = P_6 + P_7 + P_8$.
- To find each of $P_6$, $P_7$, and $P_8$ , we find the number of passwords of the specified length composed of letters and digits and subtract the number composed only of letters. We find that:

$$P_6 = 36^6 - 26^6 = 2{,}176{,}782{,}336 - 308{,}915{,}776 = 1{,}867{,}866{,}560.$$

$$P_7 = 36^7 - 26^7 =$$

$$78{,}364{,}164{,}096 - 8{,}031{,}810{,}176 = 70{,}332{,}353{,}920.$$

$$P_8 = 36^8 - 26^8 =$$

$$2{,}821{,}109{,}907{,}456 - 208{,}827{,}064{,}576 = 2{,}612{,}282{,}842{,}880.$$

Consequently, $P = P_6 + P_7 + P_8 = 2{,}684{,}483{,}063{,}360.$

العدد الكلي = عدد كلمات ملونة من 6 + ملونة من 7 + ملونة من 8

$$P = P_6 + P_7 + P_8$$

$P_6$

| 10 / 26 | 16 / 26 | | | | |
|---|---|---|---|---|---|

$$36 \times 36 \times 36 \cdots = 36^6$$

نحذف تكون كلمة السكلة حروف

حروف فقط هنوي

| 26 | 26 | 26 | 26 | 26 | 26 |
|---|---|---|---|---|---|

$$26 \times 26 \times 26 = 26^6$$

$$P_6 = 36^6 - 26^6 = 1{,}867{,}866{,}560$$

$$P_7 = 36^7 - 26^7 = 70{,}332{,}353{,}920$$

$$P_8 = 36^8 - 26^8 = 261{,}228{,}284{,}288$$

$$P = P_6 + P_7 + P_8$$

$$= 2684483063360$$

جمع n1 n2 أو or

طرح  أو or

قاعده الطرح

# The Subtraction Rule

جامعة أم القرى
UMM AL-QURA UNIVERSITY

**Subtraction Rule**: If a task can be done either in one of $n_1$ ways or in one of $n_2$ ways, then the total number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

مشتركه

- Also known as, the ***principle of inclusion-exclusion***:

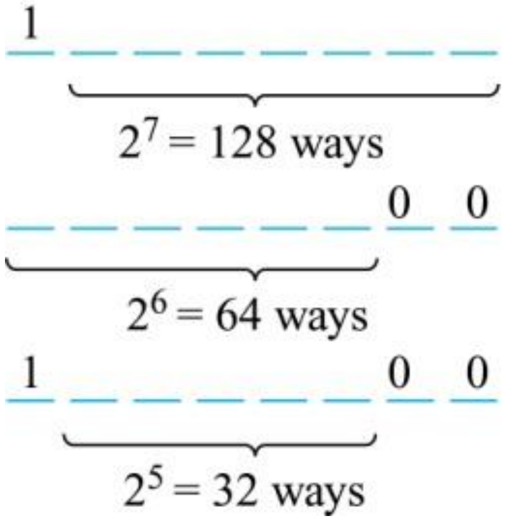
او

$$|A \cup B| = |A| + |B| - |A \cap B|$$

عدد الطرق = عدد طرق n1 + عدد طرق n2 - عدد الطرق المشتركه

# The Subtraction Rule: Examples

**Example**: How many bit strings of length eight either start with a 1 bit or end with the two bits 00?




**Solution**: Use the subtraction rule.

- Number of bit strings of length eight that start with a 1 bit: $2^7 = 128$
- Number of bit strings of length eight that end with bits 00: $2^6 = 64$
- Number of bit strings of length eight that start with a 1 bit and end with bits 00 : $2^5 = 32$

Hence, the number is $128 + 64 - 32 = 160$.

$\underset{}{\underline{1}}\ \underset{2}{\underline{1:0}}\ \underset{2}{\underline{1:0}}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}$ $\quad n_1 = 2^7$

or

$\underset{2}{\underline{1:0}}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underline{0}\ \underline{0}$ $\quad n_2 = 2^6$

$n_1 \cap n_2 = 2^5$ Common

$\underline{1}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underset{2}{\underline{\ \ }}\ \underline{0}\ \underline{0}$

$$= n_1 + n_2 - (n_1 \cap n_2)$$

$$= 2^7 + 2^6 - 2^5 = 160$$

# The Subtraction Rule: Examples

**Example:** A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business

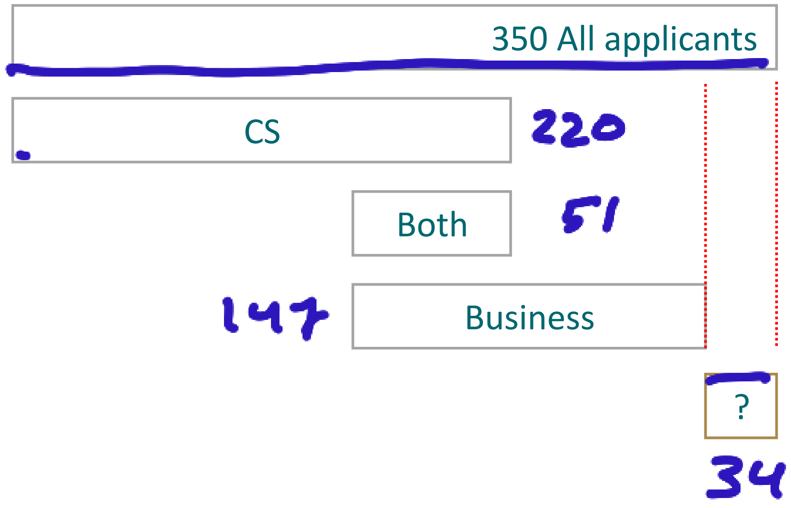
كم عدد الطلبات التي تخصها ليس CS ولا B

كم عدد الذين قدمو طلبات CS أو B

$$CS \cup B = CS + B - (CS \cap B) =$$

$$= 220 + 147 - 51 = 316$$

[Neithe CS Nor B] = 350 − 316 = 34



# The Subtraction Rule: Examples (con.)

**Solution**:

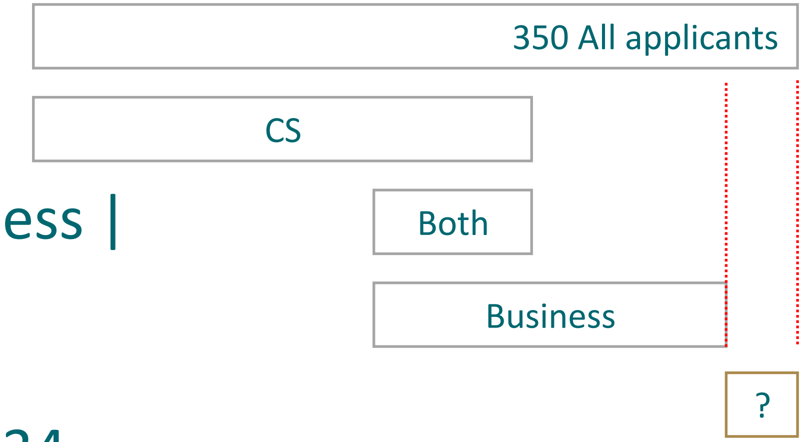
We need to find

|Neither| = |All|-|CS ∪ Business|

|CS ∪ Business| = |CS|+|Business|-| CS ∩ Business |

= 220 + 147 – 51 = 316

|Neither| = |All|-|CS ∪ Business| = 350 – 316 = 34



$n$ $W_1\ W_1\ W_1$
$W_2\ W_2\ W_2$
$W_3\ W_3\ W_3$
$\frac{n}{d}$

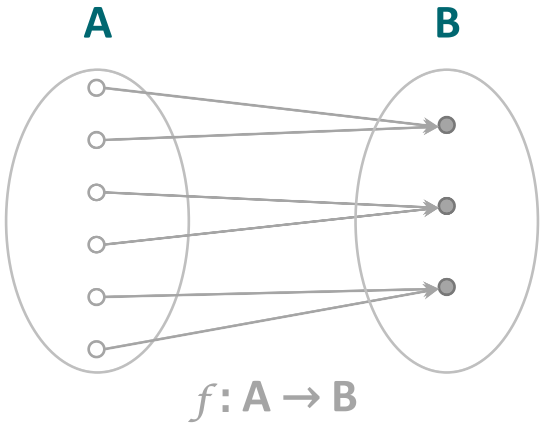
قاعدة القسمة

# The Division Rule

**Division Rule**: There are $n/d$ ways to do a task if it can be done using a procedure that can be carried out in $n$ ways, and for every way $w$, exactly $d$ of the $n$ ways correspond to way $w$. ($d$ ways are the same)

- In terms of functions: If $f$ is a function from $A$ to $B$, where both are finite sets, and for every value $y \in B$ there are exactly $d$ values $x \in A$ such that $f(x) = y$, then $|B| = |A|/d$.

$\frac{|A|}{2} = \frac{6}{2} = 3$



# The Division Rule: Examples (con.)



How many people in the house?

$n = 12$

$$\frac{n}{d} = \frac{12}{2} = 6$$

# The Division Rule: Examples (con.)

How many people in the house?

**Solution**: Each person can be counted by either their left or right shoe = 2 ways

There are 12 shoes. Therefore, by the division rule, there are 12/2 = 6 people in the house.

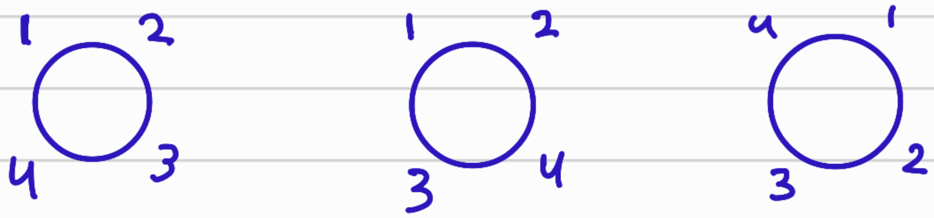
# The Division Rule: Examples

**Example**: How many ways are there to seat four people around a circular table, where two seatings are considered the same when each person has the same left and right neighbor?

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 4!$

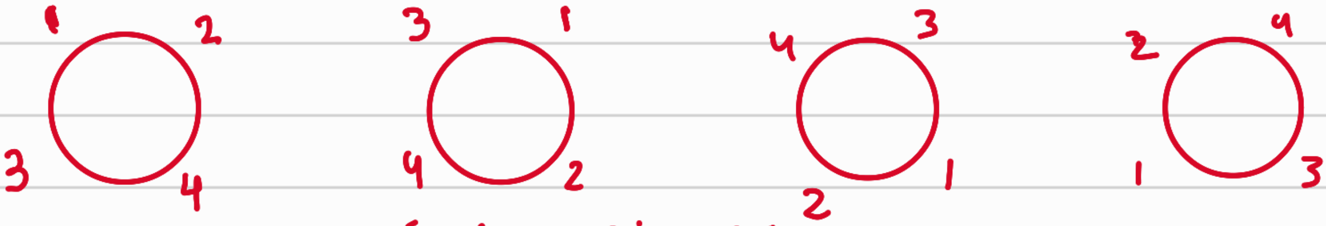
عدد طرق اجلاس ٤ اشخاص في ٤ مقاعد

$$4 \times 3 \times 2 \times 1 = 4! = 24$$



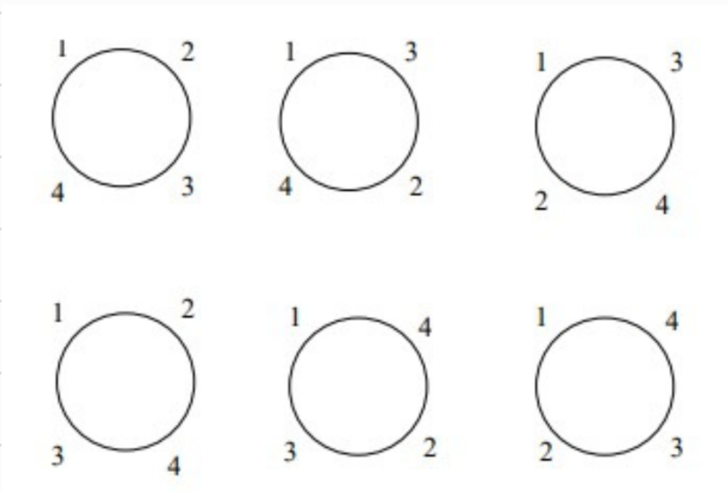
24 حالة

نلاحظ ان هذه الحالات منها تكرارات مثلاً
عندما يكون يمين ويسار نفس ، البعض نفسه تكون
نفس الحالة.



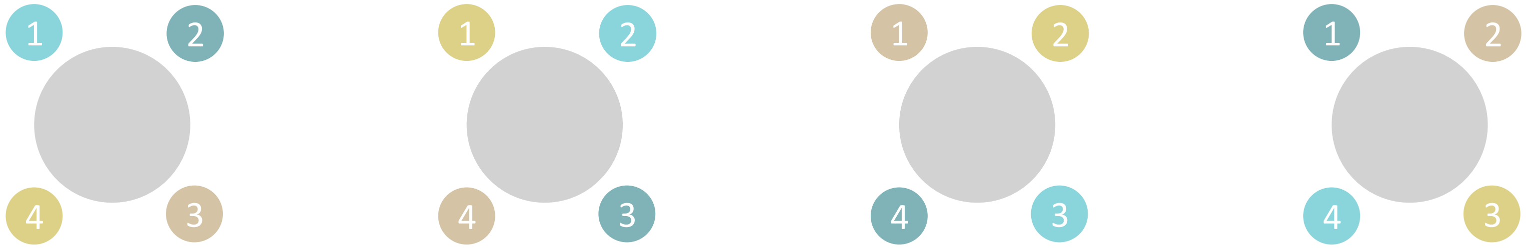
كل الحالات الاربع تمثل نفس الحالة.

يجب ان نقسم العدد الكلي $\frac{24}{4} = 6$



عدد الطرق الكلي بدون تكرار ، 6

# The Division Rule: Examples (con.)




**Solution**: Number the seats around the table from 1 to 4 proceeding clockwise.

Seat 1 = 4 people     Seat 2= 3 people     Seat 3 = 2 people     Seat 1=1 person

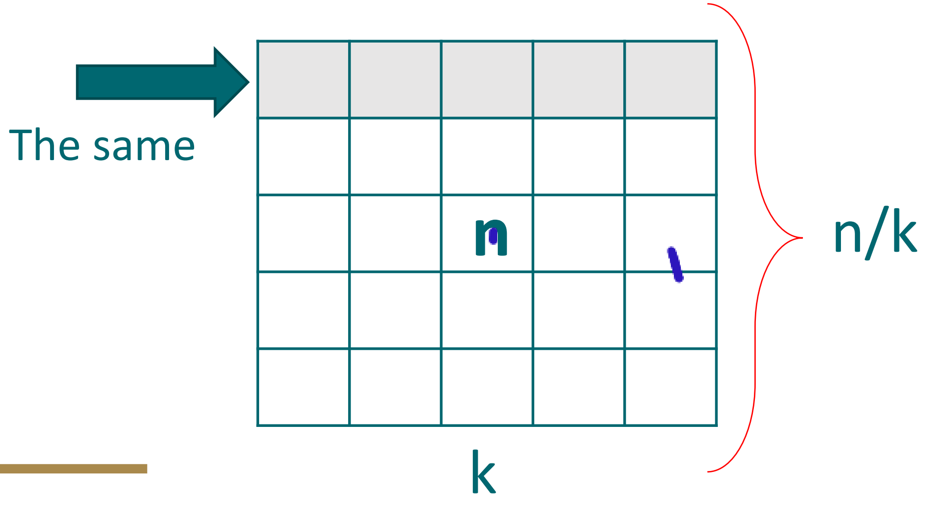
The number of ways to order 4 people = 4 * 3 * 2 * 1 = 4! = 24 ways.

Every seat has 4 choices that will have equivalent seatings. Therefore, by the division rule, there are 24/4 = 6 different seatings.

# Summary of the Counting Rules



- Sum Rule or $|A \cup B| = |A| + |B|, \quad if\ A \cap B = \emptyset$
- Subtraction Rule $|A \cup B| = |A| + |B| - |A \cap B|, \quad if\ A \cap B \neq \emptyset$
  (to get rid of the over counts)
- Product Rule $|A * B| = |A||B|$
- Division Rule Suppose we have n choices, but we want to consider some of them as "the same". If there are always exactly k choices that are the same, then the total number of "really different" choices is n/k.

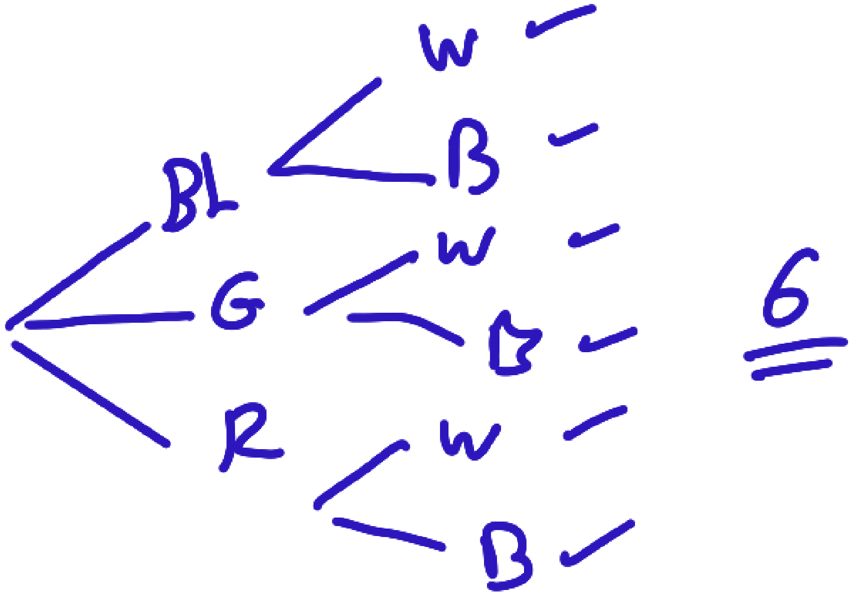


# AGENDA

- Basic Counting Principles:
  - The Product Rule
  - The Sum Rule
  - The Subtraction Rule
  - The Division Rule

- Tree Diagrams

- The Pigeonhole Principle
  - The Generalized Pigeonhole Principle



محفظ, لشجره

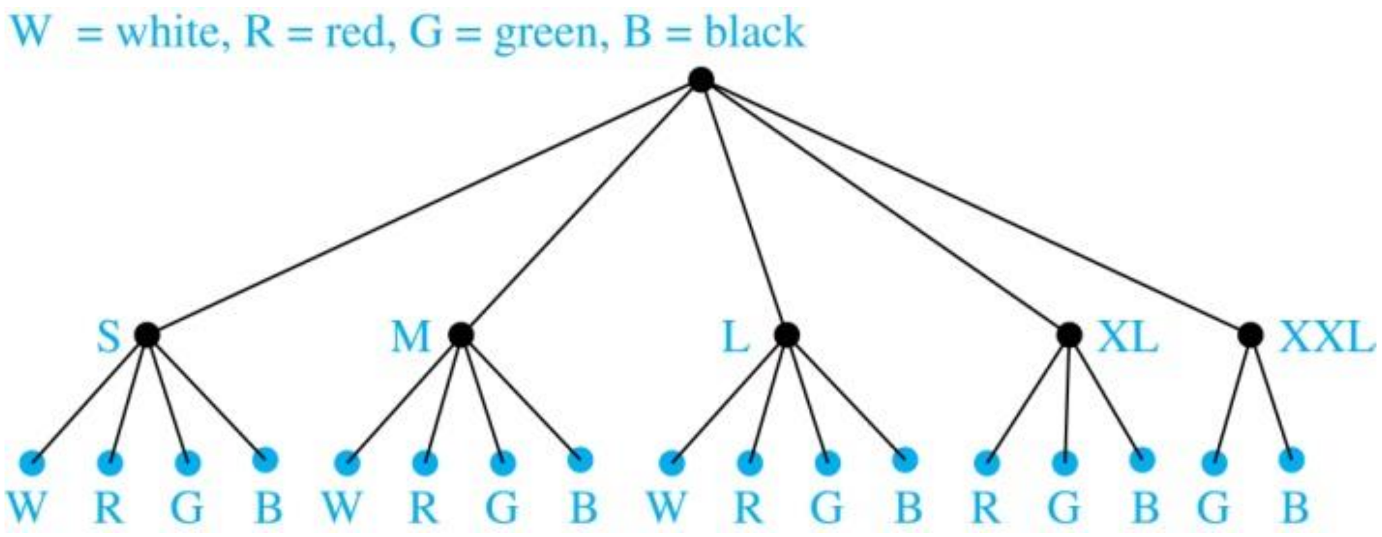
# Tree Diagrams

**Tree Diagrams**: We can solve many counting problems using *tree diagrams*, where a branch represents a possible choice, and the leaves represent possible outcomes.

**Example**: Suppose that "I Love CS" T-shirts come in five different sizes: S,M,L,XL, and XXL. Each size comes in four colors (white, red, green, and black), except XL, which comes only in red, green, and black, and XXL, which comes only in green and black. What is the minimum number of shirts that the campus book store needs to stock to have one of each size and color available?

# Tree Diagrams (con.)

**Solution**: Draw the tree diagram.




The store must stock 17 T-shirts.

# AGENDA

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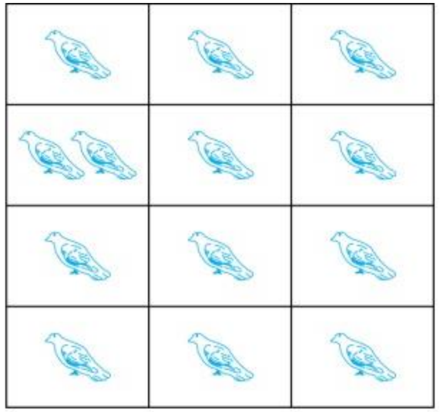
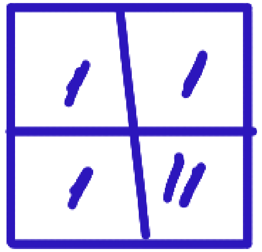
# The Pigeonhole Principle

- If a flock of 20 pigeons roosts in a set of 19 pigeonholes, one of the pigeonholes must have more than 1 pigeon.






$k = 4$

$k+1$
5

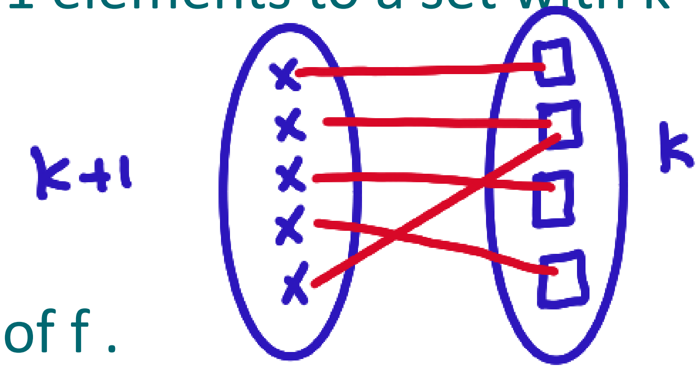
عدد الصناديق

**Pigeonhole Principle**: If k is a positive integer, and k + 1 objects are placed into k boxes, then at least one box contains two or more objects.


Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# Pigeonhole Principle: Examples


**Example**: Prove that if a function f from a set with k + 1 elements to a set with k elements is not one-to-one.




**Proof**: Use the pigeonhole principle.

- Create a box for each element y in the co-domain of f .
- Put in the box for y all of the elements x from the domain such that f(x) = y.
- Because there are k + 1 elements and only k boxes, at least one box has two or more elements.

Hence, f can’t be one-to-one.



Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

365
366

# Pigeonhole Principle: Examples

**Example**: Among any group of 367 people, there must be at least ___two___ with the same birthday, because there are only 366 possible birthdays.

366 عدد ايام اعياد الميلاد

**Example**: In any group of 27 English words, there must be at least ___two___ that begin with the same letter, because there are 26 letters in English alphabet.

عدد حروف اللغه الانجليزيه 26 حرف

| عدد الصناديق | الاشياء |
| --- | --- |
| $K$ | $K+1$ |

there is at least two object at the same box

اذا كان لدينا عدد من الصناديق k وعدد من الاشياء k+1 اذا على الاقل يوجد صندوق يحتوي 2


Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

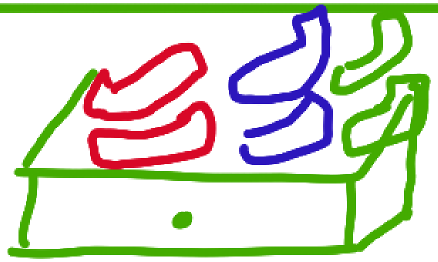
# The Generalized Pigeonhole Principle

**Example**: Among 100 people How many are there who was born in the same month?

$\frac{100}{12} = 8.3 \approx 9$

معناه بالاقل 9 اشخاص ولدوا
بنفس الشهر

كم مره على الاقل يجب ان نسحب حتى احصل على
زوج مناسب من الجرابات



$\boxed{\text{الجواب } 4}$



Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# The Generalized Pigeonhole Principle

**Example**: Among 100 people How many are there who was born in the same month?

**Solution:** Using the generalized pigeonhole principle: We assume we have 100 objects and 12 boxes. Then:
there are at least $\lceil 100/12 \rceil = 9$ people who were born in the same month.


Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

الصيغه العامه لتوزيع الحمام

# The Generalized Pigeonhole Principle

**The Generalized Pigeonhole Principle**: If N objects are placed into k boxes and N > k, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

3.5 يصير 4
2.1 يصير 3

**Notes:**

- A common type of problem asks for the minimum number of objects such that at least r objects are placed in one of the k boxes when these objects are distributed among the boxes.
- According to the PH principle, $\lceil N/k \rceil \geq r$ ;
- then:
  the smallest integer N with $N/k > r - 1$ is $\boxed{N = k(r-1)+1}$

N عدد العناصر التي ستوضع في الصناديق
K عدد الصناديق
r اقل عدد ممكن تحطه في الصناديق
at least (r)


Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

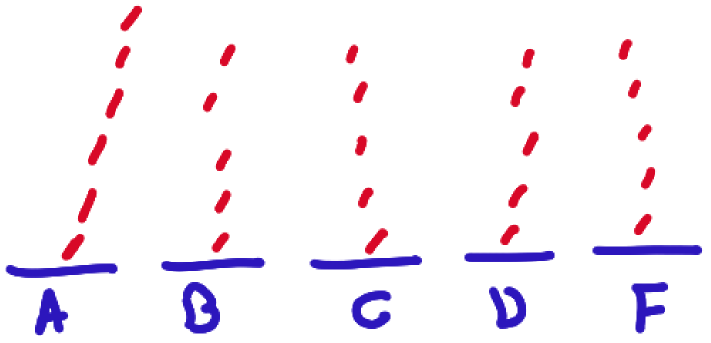
# The Generalized Pigeonhole Principle

N

**Example:** What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

الصناديق

ما اقل عدد من الطلاب لازم يحصل على الاقل 6 منهم على نفس الدرجة




اقل عدد
26

$$\boxed{\begin{aligned} N &= k(r-1)+1 \\ &= 5(6-1)+1 \\ &= 25+1 \\ &= 26 \end{aligned}}$$



Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# The Generalized Pigeonhole Principle

**Example:** What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

**Solution:**

Using pigeonhole principle, the minimum number of students needed to ensure that at least 6 students receive the same grade (in the same box) is the smallest integer N such that $\lceil N/5 \rceil = 6$.

So $N = 5(5)+1 = 26$



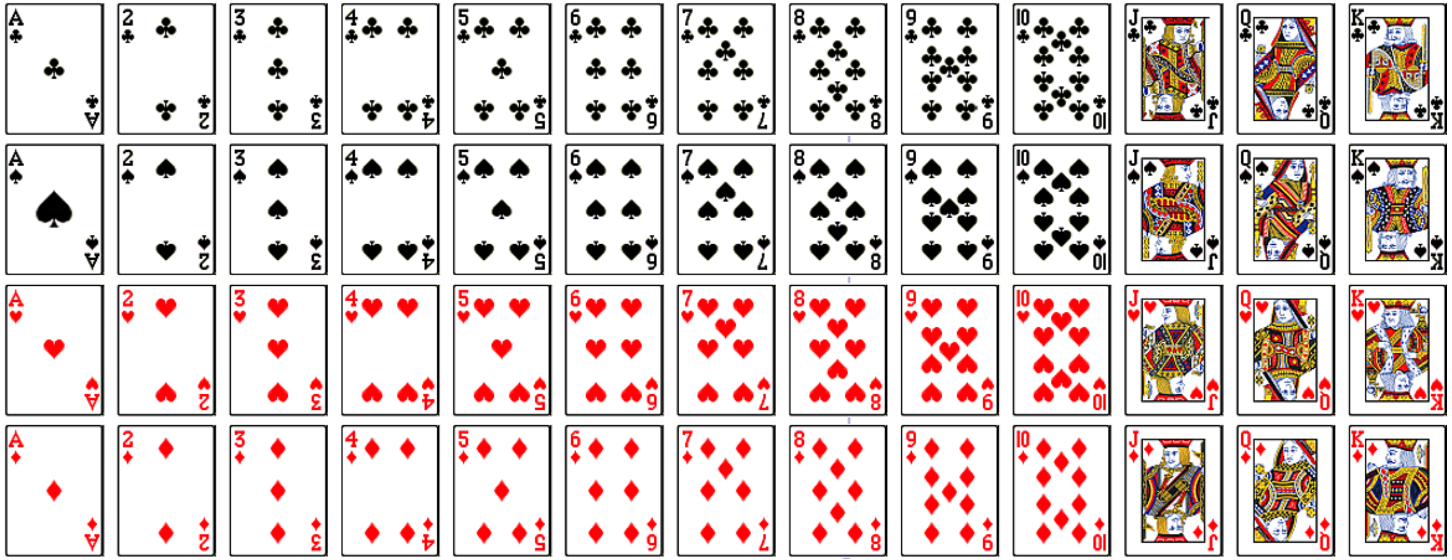
Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# The Generalized Pigeonhole Principle

**Example**: A deck of cards contains 4 suits, and each suit has 13 cards. How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

$r=3$

$$N = k(r-1)+1$$
$$= 4(3-1)+1$$
$$= 9$$





Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# The Generalized Pigeonhole Principle

**Example**: A deck of cards contains 4 suits and each suit has 13 cards.
How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

**Solution**:
We assume four boxes; one for each suit. Using the generalized pigeonhole principle, at least one box contains at least $\lceil N/4 \rceil$ cards. At least three cards of one suit are selected if $\lceil N/4 \rceil \geq 3$.

The smallest integer $N$ such that $\lceil N/4 \rceil \geq 3$ is
$N = 4(3 - 1) + 1 = 4 \cdot 2 + 1 = 9$


Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# Additional Applications for Pigeonhole Principle

- YouTube video: What Is the Pigeonhole Principle?

https://www.youtube.com/watch?v=B2A2pGrDG8I&t=2s



Acknowledgment: Slides are mainly adopted from Rosen's book supplements.

# What is Next?

- **<u>Read</u>** **Sections 6.1 and 6.2** from Rosen's Book (8th Ed).

- **<u>Solve</u>** the **Tutorial Exercises** sheet on BlackBoard (practical section) before the practical session.